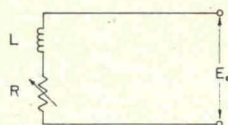


FIG. 4. Equivalent circuit of current supply when current path is through the inductor and gauge plus terminating resistors.



time constant. We will show that the error produced by this is negligible. The equivalent circuit is shown in Fig. 4 where  $E_0$  is a constant voltage source,  $L$  is an inductance, and  $R$  is the gauge plus terminating resistors. At some time,  $t=0$ , the value  $R$  will change from  $R$  to  $R+\Delta R$  representing the change in resistance due to the arrival of a stress wave. Solving the differential equation

$$LdI/dt + I(R + \Delta R) = E_0$$

with the initial conditions

$$I(0) = E_0/R = I_0$$

gives the exact solution

$$I(t) = \left( \frac{E_0}{R} - \frac{E_0}{R + \Delta R} \right) e^{-(R + \Delta R)t/L} + \frac{E_0}{R + \Delta R}$$

Assuming small time and assuming  $\Delta R$  is small compared to  $R$  give the approximate current dependence on time,

$$I(t) \approx I_0(1 - \Delta R t/L)$$

The incremental change in voltage due to the resistance change is

$$\Delta V(t) \approx I_0(1 - \Delta R t/L)\Delta R$$

The assumption of constant current implies the relation

$$\Delta V = I_0 \Delta R$$

Therefore the error in this assumption is

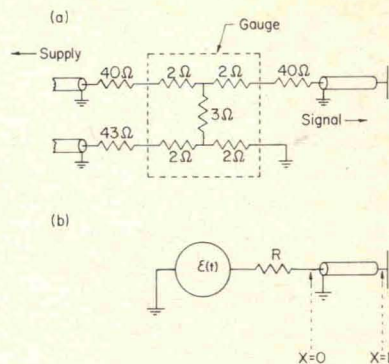
$$e = [I_0 t (\Delta R)^2] / L$$

In practice the initial current is 0.5 A, the recording time is less than 10  $\mu$ sec, the inductance is 0.3 H, and  $R$  is 100  $\Omega$ . A maximum for the change in resistance is 3  $\Omega$ . This yields an error on the order of 0.01%.

APPENDIX B

When transmission of high frequency information on coaxial cable is necessary, it is imperative that proper termination procedures are followed. The signal cable may be terminated at the oscilloscope, but this produces a shunting current that complicates the reduction of data. Figure 5(a) shows the actual manganin gauge circuit with representative values for the gauge and terminating resistors. Figure 5(b) shows the equivalent circuit. The gauge is represented by a voltage source while the gauge resistance and termination resistance are lumped into the value  $R$ .

FIG. 5. Manganin gauge circuit. (a) The actual circuit including representative gauge and terminating resistor values. (b) The equivalent circuit.



The validity of this termination technique can be seen from the following derivation.<sup>2</sup> In a lossless transmission line, both voltage and current satisfy the wave equation

$$\partial^2 I / \partial x^2 - LC \partial^2 I / \partial t^2 = 0$$

$$\partial^2 V / \partial x^2 - LC \partial^2 V / \partial t^2 = 0,$$

with the boundary conditions

$$I(L) = 0$$

and

$$V(0) + I(0)R = \epsilon(t).$$

If we adjust the gauge resistance and terminating resistor such that  $R$  is equal to the characteristic impedance of the cable,  $Z$ , the boundary conditions become

$$I(L) = 0$$

and

$$V(0) + I(0)Z = \epsilon(t).$$

The general solution for current and voltage is

$$I = f(x - ct) + g(x + ct)$$

and

$$V = Z[f(x - ct) - g(x + ct)]. \tag{1}$$

The boundary conditions give

$$f(L - ct) + g(L + ct) = 0 \tag{2}$$

and

$$2f(-ct) = \epsilon(t)/Z. \tag{3}$$

Since these conditions must hold for any argument in the domain of the solution, we have from Eq. (3)

$$f(L - ct) = \epsilon(t - L/c)/2Z \tag{4}$$

and from Eq. (2)

$$f(L - ct) = -g(L + ct). \tag{5}$$

We are concerned with the voltage at the oscilloscope. Using Eqs. (4) and (5) in Eq. (1) gives

$$V(L, t) = \epsilon(t - L/c).$$

<sup>2</sup> The author is indebted to Dr. G. E. Duvall and D. Andrews for suggesting this approach.

Thus, the voltage recorded at the oscilloscope is the voltage produced by the gauge but delayed by the transit time of the cable.

In other words, the voltage at the gauge initially sees a voltage divider consisting of the input resistance and cable impedance. It therefore transits the cable at one-half the gauge voltage. Upon arriving at the oscilloscope the pulse sees an open circuit and thus doubles its voltage propagating back along the cable as a reflected wave. The reflected wave upon reaching the gauge is properly terminated. In practice there is a problem in that  $R$  does not maintain a

constant value during the transit of the stress profile. It has been found that if the resistance is chosen such that proper termination is effected when the gauge reaches its final state value this problem is eliminated.

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